Math 350 — Homework Assignment 1, due Jan. 20, 2011

1. Consider our first skydive model given by equation (1) on page 8 in the slides. Assume we have two people jumping out of a plane, Stan, who has a mass of 65kg and a drag coefficient of 10kg/s, and Ollie, with a mass of 100kg and drag coefficient of 15kg/s. How long will it take Ollie to reach the same velocity Stan reached after 10 seconds?

2. Find the analytical solution for equation (1) in terms of a general (nonzero) initial velocity $v(0) = v_0$.

3. One refinement of the skydive model of equation (1) we did not discuss in class is to assume the drag coefficient to depend on time. In particular, use the function

$$c(t) = \begin{cases} c_1, & t < T \\ c_2, & t \geq T, \end{cases}$$

   where T denotes the time at which the skydiver opens his/her parachute, and then find the general solution assuming the initial velocity $v(0) = 0$. Hint: break the solution into two pieces.

4. (a) Determine the Taylor series for $\sinh x = \frac{e^x - e^{-x}}{2}$ about $x_0 = 0$.
   (b) Evaluate $\sinh(0.9)$ by using the first three terms of the series.
   (c) Provide an error estimate assuming you know the value of $\sinh(1) = \frac{e - 1/e}{2} = 1.175201194$. How does this error estimate compare with the true error?

5. Write the Taylor series for the function $f(x) = x^3 - 2x^2 + 4x - 1$ expanded about $x_0 = 2$, i.e., write a formula for $f(2 + h)$.

6. (a) Use the alternating series test and the triangle inequality to determine the least number of terms required to compute π as 3.14 (correctly rounded) from the series

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \ldots ?$$

   (b) Determine *experimentally* what the smallest number of terms is to get this done.
   (c) What happens if you add one more term to the expansion you used in (b)? Explain.