Math 350 — Homework Assignment 4, due March 3, 2011

1. Find the formula for the natural cubic spline that interpolates the data points $(0, 1)$, $(1, 1)$, and $(2, 5)$ and sketch the graph (or include a MATLAB plot) of the interpolating spline function for $0 \le x \le 2$.

2. Determine the coefficients in the function

$$s(x) = \begin{cases} x^3 - 1, & -9 \le x \le 0 \\ ax^3 + bx^2 + cx + d, & 0 \le x \le 5 \end{cases}$$

   so that it is a cubic spline that takes the value 2 when $x = 1$. Note we do not care about the end conditions in this problem. This problem is easiest if you stick with the representation given here and just follow a check list very similar to the one at the top of p. 30 of the slides for Chapter 3.

3. Determine the values of the parameters a, b, c, d, and e so that s is a natural cubic spline:

$$s(x) = \begin{cases} a + b(x-1) + c(x-1)^2 + d(x-1)^3, & 0 \le x < 1 \\ (x-1)^3 + ex^2 - 1, & 1 \le x \le 2. \end{cases}$$

   Hint: Similar to the previous problem, but now **with** end conditions.

4. Use the bisection method to find the *maximum* of the function

$$f(x) = -2x^6 - 1.5x^4 + 10x + 2$$

   on the interval $[0, 1]$. Perform as many iterations by hand/calculator/MATLAB as are needed for you to be able to guarantee that you have found the maximum location to a relative accuracy of 5%. Note that the MATLAB code used in class provides relative machine accuracy.

5. One of the models used in Chapter 1 gave the velocity of a falling parachutist as

$$v(t) = \frac{g\,m}{c}\left(1 - \mathrm{e}^{-(c/m)t}\right).$$

   (a) Assuming the gravitational constant g and drag coefficient c are known, find a nonlinear equation of the form $f(m) = 0$ that allows you to calculate the mass for which $v(5) = 30\text{m/s}$.

   (b) Use the the equation from (a) along with the values $g = 9.8\text{m/s}^2$ and $c = 15\text{kg/s}$ to find the mass m such that $v(5) = 30\text{m/s}$ by performing three iterations of Newton's method **by hand** with a starting value of $m_0 = 80$.

   (c) Solve the same problem with the bisection method on the interval $[60, 80]$. Iterate (also by hand) until you match the first four digits of your answer from (b), i.e., $m = 69.64\ldots$. How many iterations did it take you?