Math 350 — Homework Assignment 5, due March 24, 2011

General comment: For the small least squares problems you are solving below by hand it is fine to use the normal equations, but please remember this is not a good idea for larger (or ill-conditioned) problems.

1. Carry out four steps of the golden section search algorithm **by hand** to find

   (a) the local minimum,

   (b) the local maximum

   of the function $f(x) = 2x^3 - 9x^2 + 12x + 2$ on the interval $[0, 3]$.

2. Describe an algorithm that allows you to use the secant method to find the minimum of a given function f. Assume you are able to evaluate both f and f' at any point you need.

3. Find the constant c that best fits the following data in the least squares sense:

x	-1	2	3
y	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{5}{12}$

4. Find the best least squares fit of the form

$$f(x) = c_1 \sin \pi x + c_2 \cos \pi x$$

   for the data

x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
y	-1	0	1	2	1

5. Assume you are finding the best least squares fit by a line to the data (x_i, y_i), $i = 1, \ldots, m$, i.e., the matrix A of the overdetermined linear system $\mathsf{A}\boldsymbol{c} = \boldsymbol{y}$ is given by

$$\mathsf{A} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_m & 1 \end{bmatrix}.$$

   (a) What are the entries of the matrix $\mathsf{A}^T\mathsf{A}$ and right-hand side $\mathsf{A}^T\boldsymbol{y}$ of the normal equations?

   (b) Solve the normal equations to find formulas for the coefficients $\boldsymbol{c} = [c_1, c_2]^T$ of the fitting line $L(x) = c_1 x + c_2$ in terms of the given data.

   (c) Let

$$\overline{x} = \frac{1}{m}\sum_{k=1}^{m} x_k, \quad \overline{y} = \frac{1}{m}\sum_{k=1}^{m} y_k, \quad \hat{x} = \sum_{k=1}^{m}(x_k - \overline{x})^2,$$

   and verify that the formulas

$$\begin{aligned} c_1 &= \frac{1}{\hat{x}}\sum_{k=1}^{m}(x_k - \overline{x})(y_k - \overline{y}) \\ c_2 &= \overline{y} - c_1\overline{x} \end{aligned}$$

   are equivalent to those found in part (b).