Math 350 — Homework Assignment 6, due April 7, 2011

1. Approximate the value of the integral
$$\int_{-1}^{1} \left| x - \frac{1}{2} \right| dx$$
using L_n, R_n, and T_n for $n = 1, 2, 4, 8$ intervals of equal length. Account for the differences in the results and compare with the exact solution.

2. Derive the basic Simpson's rule by integrating a quadratic interpolating polynomial over the intervals $[x_{i-1}, x_i] \cup [x_i, x_{i+1}]$ as indicated in Chapter 6 of the classnotes. Assume that the integration nodes are equally spaced.

3. Consider a two-term numerical integration rule of the form
$$\int_{-1}^{1} f(x)dx \approx w_1 f(x_1) + w_2 f(x_2).$$
Determine the weights w_1 and w_2 and the nodes x_1 and x_2 so that the above rule is *exact* for the functions $f(x) = 1$, $f(x) = x$, $f(x) = x^2$, and $f(x) = x^3$. This means you have to solve a system of 4 *nonlinear* equations in four unknowns. Feel free to use any method of your choice to do this (by hand, with Mathematica, Maple or MATLAB's Symbolic Toolbox, with a numerical method such as Newton's method, etc.).