Math 350 — Midterm, March 12, 2008

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1. (a) Solve the following linear system by LU factorization/Gauss elimination with pivoting:

$$\begin{aligned} 2x + 6y + 10z &= 0 \\ x + 3y + 3z &= 2 \\ 3x + 14y + 28z &= -8. \end{aligned}$$

   Show and explain all the details of your work.

   (b) What do the matrices P, L and U look like?

2. Here are a matrix A and its L and U factors:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 5 \\ 0 & 0 & 8 \end{bmatrix}.$$

(a) Use this information to compute $\det(A)$.

(b) Use this information to compute A^{-1}. Do not compute any matrix inverses – solve linear systems!

3. Find the Lagrange form of the interpolating polynomial for the data

x	-2	0	2
y	4	2	8

.

4. Determine whether the function

$$S(x) = \begin{cases} 10 - 24x + 18x^2 - 4x^3, & 1 \le x \le 2 \\ -54 + 72x - 30x^2 + 4x^3, & 2 \le x \le 3 \end{cases}$$

is a cubic spline on $[1, 3]$.

5. Let $f(x) = x^3 - 4$.

   (a) Apply four iterations of the bisection method on $[a, b] = [0, 2]$ to approximate a root of f.

   (b) Use Newton's method with $x_0 = 1$ to compute x_3.

6. Recall that Newton's method for systems of nonlinear equations is given by

> Input $\boldsymbol{f}$, J, $\boldsymbol{x}^{(0)}$
>
> for $n = 0, 1, 2, \ldots$ do
>
>   Solve $J(\boldsymbol{x}^{(n)})\boldsymbol{h} = -\boldsymbol{f}(\boldsymbol{x}^{(n)})$ for $\boldsymbol{h}$
>
>   Update $\boldsymbol{x}^{(n+1)} = \boldsymbol{x}^{(n)} + \boldsymbol{h}$
>
> end
>
> Output $\boldsymbol{x}^{(n+1)}$

where the Jacobian is of the form

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_m} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_m} \end{bmatrix}.$$

Perform one iteration of Newton's method starting with $\boldsymbol{x}^{(0)} = [\pi/4, \pi/4]^T$ to get an approximate solution of the system

$$\begin{aligned} 5\cos(\alpha) + 6\cos(\alpha + \beta) &= 10 \\ 5\sin(\alpha) + 6\sin(\alpha + \beta) &= 4. \end{aligned}$$

7. Give one possible MATLAB code fragment that would allow you to define the matrix

$$A = \begin{bmatrix} -10 & 1 & 4 & 0 & 0 & 0 \\ 1 & -10 & 0 & 4 & 0 & 0 \\ 4 & 0 & -10 & 1 & 4 & 0 \\ 0 & 4 & 1 & -10 & 0 & 4 \\ 0 & 0 & 4 & 0 & -10 & 1 \\ 0 & 0 & 0 & 4 & 1 & -10 \end{bmatrix}$$

in sparse matrix format.