MATH 461 – Fourier Series and Boundary-Value Problems

Time and Location: 1:50pm–3:05pm TR, Location E1 026
Instructor: Greg Fasshauer
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Office hours: TR: 12:30pm–1:30pm, also by appointment
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Office hours: M 3:00pm–4:30pm, F 12:00pm–1:30pm, E1 129

Textbook: R. Haberman, *Applied Partial Differential Equations*, 5th ed., Pearson (2012), ISBN 032179706X.

Other required material: I will occasionally use software such as MATLAB or Mathematica

Prerequisites: MATH 251, MATH 252

Objectives:

1. Students will understand how PDEs (such as the heat and wave equation) model physical phenomena and the basic structure of PDEs and their solutions.
2. Students will learn the separation of variable technique for solving linear second-order PDEs (heat equation, Laplace's equation, wave equation).
3. Students will learn the basics of Fourier series.
4. Students will learn the basic properties of and how to solve regular Sturm-Liouville eigenvalue problems.
5. Students will learn to solve some partial differential equations in more than one space variable.

Course Outline: Hours

1. Heat Equation — 6
   a. Derivation in 1D
   b. Different types of boundary conditions
   c. Derivation in 2D/3D
2. Separation of Variables — 9
   a. Linearity
   b. Eigenvalues and eigenfunctions
   c. Orthogonality of functions
   d. Separation of variables for heat equation in 1D with various boundary conditions
   e. Separation of variables for Laplace's equation in rectangular and circular domains
   f. Maximum principle for Laplace's equation and well-posedness of Laplace's equation
3. Fourier Series — 9
   a. Piecewise smooth and periodic functions
   b. Convergence of Fourier series

    c. Fourier sine and cosine series
    d. Term-by-term differentiation and integration of Fourier series
    e. Complex form of Fourier series
4. Vibrating Strings and Membranes 3
    a. Derivation of 1D wave equation
    b. Boundary conditions
    c. Separation of variables
5. Sturm-Liouville Eigenvalue Problems 12
    a. Motivating examples (e.g., heat equation for non-uniform rod)
    b. Regular Sturm-Liouville problems and their properties (generalized Fourier series, self-adjointness, Green's formula, orthogonality of eigenfunctions, Rayleigh quotient)
    c. Wave equation for non-uniform strings
    d. Rayleigh-Ritz principle and approximation properties
6. Partial Differential Equations in Space 3
    a. Membranes of arbitrary shape
    b. Wave equation in arbitrary regions
    c. Wave equation for circular membranes and Bessel functions

Assessment:	Homework	20%
	Midterms (Oct.6 & Nov.17)	50%
	Final Exam (Dec.9, 8:00am–10:00am)	30%

Homework: There will be frequent homework assignments in the form of written reports. While I do encourage study groups and team learning, I expect that homework solutions are written up by each person **individually**. **Duplicate solutions** will be considered evidence of academic dishonesty.

Exams: Hour exams are tentatively scheduled as listed above. The final will be comprehensive, and approximately 75-80% of the problems will be taken from the material covered in the midterms. **Make-ups for tests will be given only when authorized in advance** by the instructor. Only 90% of the make-up score will count.

Academic Integrity: Unless otherwise indicated by the instructor, this course follows the Academic Integrity Policy of IIT's College of Science. This implies that

- students are aware of and follow the IIT Code of Academic Honesty https://web.iit.edu/student-affairs/handbook/fine-print/code-academic-honesty
- a violation of academic integrity will result in a reduced course grade,
- any violation of academic integrity will be reported to the Office of the Vice Provost for Academic Affairs (academichonesty@iit.edu).

Students with Disabilities: Reasonable accommodations will be made for students with documented disabilities. In order to receive accommodations, students must obtain a letter of accommodation from the Center for Disability Resources and make an appointment to speak with me as soon as possible. The Center for Disability Resources is located at 3424 S. State Street - 1C3-2, 312-567-5744 or disabilities@iit.edu.