Math 472: Assignment 1 — due Wednesday, Sept. 14, 2005

1. Show that the function $f(t, x) = x^2 e^{-t^2} \sin t$ is Lipschitz continuous for $x \in [0, 2]$.

2. (a) Approximate the function $f(x) = e^{x/2}$ over the interval $[1, 9]$ by a fourth-degree polynomial in two ways: using a Taylor polynomial centered at $\xi = 5$, and using the Lagrange form of the interpolating polynomial with $\xi_0 = 1$, $\xi_1 = 3$, $\xi_2 = 5$, $\xi_3 = 7$, and $\xi_4 = 9$.
(b) Plot the error estimates for these two approaches (using Taylor's Theorem and the Lagrange form of the interpolating polynomial) for $x \in [0, 12]$.
(c) Plot the actual error for these approximants on $[0, 12]$. Comment.

3. Use the Peano kernel theorem to obtain the following well-known formula for *Simpson's rule*:

$$\int_0^2 f(x)dx = \frac{1}{3}\left[f(0) + 4f(1) + f(2)\right] - \frac{1}{90} f^{(4)}(\xi).$$

4. (a) Write the following system of initial value problems

$$\begin{aligned} y'' + yz &= 0, \quad y(0) = 1, \quad y'(0) = 0 \\ z' + 2yz &= 4, \quad z(0) = 3 \end{aligned}$$

as a system of first-order initial value problems.
(b) Convert the following system of higher-order time-dependent ODEs into a system of first-order equations that do not explicitly depend on t:

$$\begin{aligned} x''' - 5tx''y'' + \ln(x')z &= 0 \\ y'' - \sin(ty) + 7tx'' &= 0 \\ z' + 16ty' - e^t zx' &= 0. \end{aligned}$$

Hint: introduce an additional differential equation for t.