1. Show that the function \( f(t, x) = x^2 e^{-t^2} \sin t \) is Lipschitz continuous for \( x \in [0, 2] \).

2. (a) Approximate the function \( f(x) = e^{x/2} \) over the interval \([1, 9]\) by a fourth-degree polynomial in two ways: using a Taylor polynomial centered at \( \xi = 5 \), and using the Lagrange form of the interpolating polynomial with \( \xi_0 = 1, \xi_1 = 3, \xi_2 = 5, \xi_3 = 7, \) and \( \xi_4 = 9 \).
   (b) Plot the error estimates for these two approaches (using Taylor’s Theorem and the Lagrange form of the interpolating polynomial) for \( x \in [0, 12] \).
   (c) Plot the actual error for these approximants on \([0, 12]\). Comment.

3. Use the Peano kernel theorem to obtain the following well-known formula for Simpson’s rule:

   \[
   \int_0^2 f(x)dx = \frac{1}{3} [f(0) + 4f(1) + f(2)] - \frac{1}{90} f^{(4)}(\xi).
   \]

4. (a) Write the following system of initial value problems

   \[
   \begin{align*}
   y'' + yz &= 0, \quad y(0) = 1, \quad y'(0) = 0 \\
   z' + 2yz &= 4, \quad z(0) = 3 
   \end{align*}
   \]

   as a system of first-order initial value problems.

   (b) Convert the following system of higher-order time-dependent ODEs into a system of first-order equations that do not explicitly depend on \( t \):

   \[
   \begin{align*}
   x''' - 5tx''y' + \ln(x')z &= 0 \\
   y'' - \sin(ty) + 7tx'' &= 0 \\
   z' + 16ty' - e^t xx' &= 0.
   \end{align*}
   \]

   Hint: introduce an additional differential equation for \( t \).