Math 472: Assignment 6 — due Monday, Nov. 14, 2005

1. Verify that the function $y(t) = c \sin t$ is a solution of the boundary value problem

$$y''(t) + y(t) = 0$$
$$y(0) = 0, \quad y(\pi) = 0$$

for any constant c. Comment.

2. Find the solution at $t = \frac{1}{2}$ of the linear two-point boundary value problem

$$y''(t) + 2y'(t) + 10t = 0$$
$$y(0) = 1, \quad y(1) = 2$$

by applying the finite difference method (by hand) with $h = \frac{1}{2}$.

3. Consider the linear boundary value problem

$$y''(t) = u(t) + v(t)y(t) + w(t)y'(t)$$
$$a_0 y(a) + a_1 y'(a) = \alpha, \quad b_0 y(b) + b_1 y'(b) = \beta.$$

Set up the resulting system of linear equations if the finite difference method is used with meshsize $h = \frac{b-a}{m+1}$. Make sure that you use only $\mathcal{O}(h^2)$ approximations.