Math 472: Computer Assignment 4 — due Monday, Nov.21, 2005

1. Write a Matlab function `[t,y] = bvpsolve(u,v,w,a,b,alpha,beta,m)` to solve a linear two-point boundary value problem of the form

$$y''(t) = u(t) + v(t)y(t) + w(t)y'(t)$$
$$y(a) = \alpha, \quad y(b) = \beta$$

   with the finite difference method. Use the subroutine `tridiag.m` presented in class to solve the tridiagonal linear system.

   Assume that the functions u, v and w are defined separately, e.g., in a driver script.

2. Consider the problem

$$t^2 y''(t) - t(t+2)y'(t) + (t+2)y(t) = 0$$

   whose general solution is given by $y(t) = c_1 t + c_2 t e^t$.

   (a) What is the solution if the boundary conditions

$$y(1) = e, \quad y(2) = 2e^2$$

   are used?

   (b) Test your code from Exercise 1 with this problem. Plot the approximate and exact solutions together for $m = 19$.

   (c) Perform a series of experiments with $m = 4, 9, 19, 39, 79$, compute the maximum error and observe how it changes with m (or h).

3. Repeat Exercise 2 for the problem

$$y''(t) + 2y'(t) + 10t = 0$$
$$y(0) = 1, \quad y(1) = 2$$

   whose general solution is given by $y(t) = -\frac{5}{2}t^2 + \frac{5}{2}t + c_1 e^{-2t} + c_2$.