Math 477 — Homework Assignment 2, due Sept.28, 2006

1. Determine SVDs of the following matrices. Do not use a computer, and do not use the method for hand calculations discussed in class. Use only basic properties of the SVD and note that the matrices are either diagonal matrices or rank-1 matrices:

$$\text{(a)} \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}, \quad \text{(b)} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \quad \text{(c)} \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{(d)} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad \text{(e)} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

2. In the discussion of matrix norms we claimed that the 2-norm of the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

   is approximately 1.6180. Using the SVD, work out (the "by-hand" method is from now on allowed) the exact values of $\sigma_{\min}(A)$ and $\sigma_{\max}(A)$ for this matrix.

3. Find the SVDs of the following matrices:

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 5 \\ -4 \end{bmatrix}.$$

4. If P is an orthogonal projector, then $I - 2P$ is unitary. Prove this algebraically, and give a geometric interpretation.

5. Consider the matrices

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

   Answer the following questions by hand calculation.

   (a) What us the orthogonal projector P onto range(A), and what is the image under P of the vector $[1, 2, 3]^*$?

   (b) Same questions for B.