

Math 478/578: Computer Assignment 2 — due Tuesday, Feb. 27, 2007

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- Using various values of  $\lambda$ , such as 5, -5, or -10, numerically solve the following initial value problem using the fourth-order Runge-Kutta method:

$$\begin{aligned}y'(t) &= \lambda y(t) + \cos t - \lambda \sin t \\y(0) &= 0.\end{aligned}$$

Use step size  $h = 0.01$ . Compare the numerical solution to the analytic solution on the interval  $[0, 5]$  using both a plot and the maximum norm of the error vector computed on a fine evaluation grid of 1000 uniformly spaced points. What effect does  $\lambda$  have on the numerical accuracy?

- In the 1968 Olympic games in Mexico City, Bob Beamon established a world record with a long jump of 8.90 meters. This was 0.80 meters longer than the previous world record. Since 1968, Beamon's jump has been exceeded only once in competition, by Mike Powell's jump of 8.95 meters in Tokyo in 1991. After Beamon's remarkable jump, some people suggested that the lower air resistance at Mexico City's 7400 ft. altitude was a contributing factor. This problem examines that possibility.

For the mathematical model we assume a fixed Cartesian coordinate system with horizontal  $x$ -axis, vertical  $y$ -axis, and its origin at the takeoff board. The jumper's initial velocity has magnitude  $v_0$  and makes an angle with respect to the  $x$ -axis of  $\theta_0$  radians. The only forces acting after takeoff are gravity and the aerodynamic drag,  $D$ , which is proportional to the square of the magnitude of the velocity. There is no wind. The equations describing the jumper's motion are

$$\begin{aligned}x'(t) &= v(t) \cos \theta(t), & y'(t) &= v(t) \sin \theta(t) \\ \theta'(t) &= -\frac{g}{v(t)} \cos \theta(t), & v'(t) &= -\frac{D}{m} - g \sin \theta(t).\end{aligned}$$

The drag is

$$D = \frac{c\rho s}{2} (x'(t)^2 + y'(t)^2).$$

Constants for this exercise are the acceleration gravity,  $g = 9.81\text{m/s}^2$ , the mass,  $m = 80\text{kg}$ , the drag coefficient,  $c = 0.72$ , the jumper's cross-sectional area,  $s = 0.50\text{m}^2$ , and the take-off angle,  $\theta_0 = 22.5^\circ = \pi/8$  radians.

Use one of Matlab's built-in ODE solvers to compute four different jumps, with different values for initial velocity,  $v_0$ , and air density,  $\rho$ . The length of each jump is  $x(t_f)$ , where the air time,  $t_f$ , is determined by the condition  $y(t_f) = 0$ .

- "Nominal" jump at high altitude.  $v_0 = 10\text{m/s}$ , and  $\rho = 0.94\text{kg/m}^3$ .
- "Nominal" jump at sea level.  $v_0 = 10\text{m/s}$ , and  $\rho = 1.29\text{kg/m}^3$ .
- Sprinter's approach at high altitude.  $\rho = 0.94\text{kg/m}^3$ . Determine  $v_0$  so that the length of the jump is Beamon's record, 8.90 m.
- Sprinter's approach at sea level.  $\rho = 1.29\text{kg/m}^3$ , and  $v_0$  is the value determined in (c).

Present your results by completing the following table.

v0	theta0	rho	distance
10.0000	0.3927	0.9400	???
10.0000	0.3927	1.2900	???
???	0.3927	0.9400	8.9000
???	0.3927	1.2900	???

Which is more important, the air density or the jumper's initial velocity?