

Math 478 — Homework Assignment 4, due March 8, 2007

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1. By considering the scalar equation  $y'(t) = f(t)$ , i.e.,  $f$  is independent of  $y$ , show that in this case the classical fourth-order Runge-Kutta method is equivalent to *Simpson's rule*

$$\int_a^b f(x)dx \approx \frac{b-a}{6} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right].$$

2. Determine all values of  $\theta$  such that the theta method

$$\mathbf{y}_{n+1} = \mathbf{y}_n + h [\theta \mathbf{f}(t_n, \mathbf{y}_n) + (1 - \theta) \mathbf{f}(t_{n+1}, \mathbf{y}_{n+1})]$$

is  $A$ -stable.

3. Is there any reason to distrust the following numerical scheme for solving the IVP  $\mathbf{y}' = \mathbf{f}(t, \mathbf{y})$

$$\mathbf{y}_{n+3} - 3\mathbf{y}_{n+2} + 2\mathbf{y}_{n+1} = h [\mathbf{f}_{n+3} + 2\mathbf{f}_{n+2} + \mathbf{f}_{n+1} - 2\mathbf{f}_n]?$$

Explain.