Math 578 — Homework Assignment 3, due Feb. 22, 2007

1. Derive explicitly the three-step Adams-Moulton and Adams-Bashforth methods.

2. Which of the following multistep methods is convergent?

   (a) $\boldsymbol{y}_{n+2} - \boldsymbol{y}_n = h\,[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) - 3\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1}) + 4\boldsymbol{f}(t_n, \boldsymbol{y}_n)]$,

   (b) $\boldsymbol{y}_{n+2} - 2\boldsymbol{y}_{n+1} + \boldsymbol{y}_n = h\,[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) - \boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1})]$,

   (c) $\boldsymbol{y}_{n+2} - \boldsymbol{y}_{n+1} - \boldsymbol{y}_n = h\,[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) - \boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1})]$,

   (d) $\boldsymbol{y}_{n+2} - 3\boldsymbol{y}_{n+1} + 2\boldsymbol{y}_n = h\,[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) + \boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1})]$,

   (e) $\boldsymbol{y}_{n+2} - \boldsymbol{y}_n = h\,[\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) - 3\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1}) + 2\boldsymbol{f}(t_n, \boldsymbol{y}_n)]$,

3. Determine the order of the three-step method

$$\boldsymbol{y}_{n+3} - \boldsymbol{y}_n = h\left[\frac{3}{8}\boldsymbol{f}(t_{n+3}, \boldsymbol{y}_{n+3}) + \frac{9}{8}\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) + \frac{9}{8}\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1}) + \frac{3}{8}\boldsymbol{f}(t_n, \boldsymbol{y}_n)\right],$$

   the *three-eighths* scheme. Is it convergent?

4. Show that the multistep method

$$\boldsymbol{y}_{n+3} + \alpha_2\boldsymbol{y}_{n+2} + \alpha_1\boldsymbol{y}_{n+1} + \alpha_0\boldsymbol{y}_n = h\,[\beta_2\boldsymbol{f}(t_{n+2}, \boldsymbol{y}_{n+2}) + \beta_1\boldsymbol{f}(t_{n+1}, \boldsymbol{y}_{n+1}) + \beta_0\boldsymbol{f}(t_n, \boldsymbol{y}_n)]$$

   is fourth order only if $\alpha_0 + \alpha_2 = 8$ and $\alpha_1 = -9$. Hence deduce that this method cannot be both fourth order and convergent.