# Math 590 – Meshfree Methods

**Time and Location**: 11:25am–12:40pm MW, Location E1 027
**Instructor**: Greg Fasshauer
**Office**: 208A E1
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**Office hours**: MW: 2:00pm–3:00pm, also by appointment

**Textbook(s):** G. Fasshauer, *Meshfree Approximation Methods with MATLAB*, World Scientific, 2007.
H. Wendland, *Scattered Data Approximation*, Cambridge University Press, 2004.

**Other required material:** MATLAB

**Prerequisites:** Some exposure to computational mathematics and advanced analysis, consent of the instructor.

**Objectives:**

1. Students will learn the definitions and understand the key concepts of multivariate scattered data approximation with kernel-based methods,
2. Students will learn to solve multivariate interpolation and least squares approximation problems,
3. Students will learn how to apply these methods to the solution of partial differential equations,
4. Students will learn how to implement and use these algorithms in MATLAB,
5. Students will improve their problem solving skills in computational mathematics,
6. Students will improve their presentation and writing skills.

**Course Outline:**

| | | Hours |
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| 1. | Introduction including a historical perspective | 2 |
| 2. | Scattered data fitting and the Haar-Mairhuber-Curtis theorem | 2 |
| 3. | Positive definite kernels and reproducing kernel Hilbert spaces | 6 |
| 4. | Examples of kernels, including radial, non-radial and anisotropic kernels | 2 |
| 5. | Connection to kriging | 4 |
| 6. | Connection to Green's kernels | 2 |
| 7. | Generalized Sobolev spaces | 2 |
| 8. | Accuracy and optimality of RKHS methods | 2 |
| 9. | "Flat" limits | 1 |
| 10. | The uncertainty principle – an unfortunate misconception | 1 |
| 11. | Alternate bases (data-dependent: Lagrange, Newton; data-independent: eigenfunctions) | 3 |
| 12. | Stable computation, Hilbert-Schmidt SVD | 4 |
| 13. | Parameter optimization, MLE, cross validation | 3 |
| 14. | Machine learning (RBF networks and SVMs) | 2 |
| 15. | Hermite interpolation, approximation of derivative data | 2 |
| 16. | Various methods for PDEs (collocation, MFS, MPS, method of lines) | 4 |

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| **Assessment**: | Homework | 25% |
| | Computer Programs | 20% |
| | Test | 25% |
| | Final Exam/Project (Tues., Dec.9, 8am-10am) | 30% |