Math 532 — Homework 12 — Due: Wednesday, April 29, 2015

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1. Do Exercise 7.3.7 in the textbook. Hint: Use the Cayley–Hamilton theorem.

2. Do Exercise 7.5.6 in the textbook. However, I will not accept multiples of the identity matrix as an example. Instead, describe a general strategy for constructing an example of the desired type and then use it to give a specific example.

3. To investigate what happens with convergence of the power method when there is no single dominant eigenvalue consider the matrices

$$\mathsf{A} = \begin{pmatrix} 0 & 6 & 3 \\ -1 & 5 & 1 \\ -1 & 2 & 4 \end{pmatrix}, \quad \mathsf{B} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ 0 & 2 & 1 \end{pmatrix}, \quad \mathsf{C} = \begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -4 \\ 0 & 6 & 5 \end{pmatrix},$$

with eigenvalues

$$\begin{aligned} &\sigma(\mathsf{A}) = \{3\}, \quad \text{with } \mathrm{algmult}_{\mathsf{A}}(3) = 3, \\ &\sigma(\mathsf{B}) = \{1, 0, -1\}, \quad \text{with } \mathrm{algmult}_{\mathsf{B}}(1) = \mathrm{algmult}_{\mathsf{B}}(0) = \mathrm{algmult}_{\mathsf{B}}(-1) = 1, \\ &\sigma(\mathsf{C}) = \{1, -1\}, \quad \text{with } \mathrm{algmult}_{\mathsf{C}}(1) = 2 \text{ and } \mathrm{algmult}_{\mathsf{C}}(-1) = 1, \end{aligned}$$

and eigenspaces

$$\begin{aligned} &N(\mathsf{A} - 3\mathsf{I}) = \mathrm{span}\{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\}, \\ &N(\mathsf{B} - \mathsf{I}) = \mathrm{span}\{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\}, \quad N(\mathsf{B}) = \mathrm{span}\{\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}\}, \quad N(\mathsf{B} + \mathsf{I}) = \mathrm{span}\{\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\}, \\ &N(\mathsf{C} - \mathsf{I}) = \mathrm{span}\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\}, \quad N(\mathsf{C} + \mathsf{I}) = \mathrm{span}\{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\}. \end{aligned}$$

In all three cases, perform several iterations of the power method with initial vector $\boldsymbol{x}_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and report what happens.

While this problem is best performed using software such as Matlab or R, the numbers are (almost?) nice enough so that (tedious) hand calculations should also be possible. It is highly recommended that $\boldsymbol{x}$ (and $\boldsymbol{x}_0$) are normalized before multiplication by A (see slide #71 in Chapter 7).