Math 532 — Homework 5 — Due: Wednesday, February 18, 2015

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1. Here is what we used on slides 89–90 of Chapter 4: Let A be an $m \times n$ matrix and B be $n \times p$. Show that

   (a) $R(\mathsf{AB}) \subseteq R(\mathsf{A})$.

   (b) $N(\mathsf{B}) \subseteq N(\mathsf{AB})$

2. Let A be an $m \times n$ matrix and B be $n \times p$. Show that

$$\dim(N(\mathsf{AB})) = \dim(N(\mathsf{B})) + \dim(N(\mathsf{A}) \cap R(\mathsf{B})).$$

3. Let

$$\mathsf{A} = \begin{pmatrix} 1 & 2.01 \\ -1 & -2 \\ 3 & 6 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1.01 \\ -1 \\ 3 \end{pmatrix}$$

   (a) Determine rank(A) and solve $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$ using exact arithmetic.

   (b) Determine rank($\mathsf{A}^T\mathsf{A}$) and solve $\mathsf{A}^T\mathsf{A}\boldsymbol{x} = \mathsf{A}^T\boldsymbol{b}$ using exact arithmetic.

   (c) Use 3-digit arithmetic to find rank(A) and solve $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$.

   (d) Use 3-digit arithmetic to find $\mathsf{A}^T\mathsf{A}$ and $\mathsf{A}^T\boldsymbol{b}$ and then solve $\mathsf{A}^T\mathsf{A}\boldsymbol{x} = \mathsf{A}^T\boldsymbol{b}$.

4. Use an exponential model of the form $f(t) = \alpha e^{\beta t}$ to obtain a least squares fit for the data

t	1	2	3	4	5
b	16	27	45	74	122

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5. Do Exercise 4.6.8 in the textbook.