Math 532 — Homework 7 — Due: Wednesday, March 4, 2015

1. Show that the norm $\|\cdot\|$ is a function that depends continuously on its argument $\boldsymbol{x} \in \mathbb{R}^n$, i.e., for every $\epsilon > 0$ there exists a δ such that
$$\left|\, \|\boldsymbol{x}\| - \|\boldsymbol{y}\| \,\right| < \epsilon$$
whenever $|x_i - y_i| < \delta$ for each $i = 1, \ldots, n$.

2. Let $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ be vectors in $\mathbb{R}^n$. Prove that
$$\|\boldsymbol{z} - \boldsymbol{x}\|_2^2 + \|\boldsymbol{z} - \boldsymbol{y}\|_2^2 = \frac{1}{2}\|\boldsymbol{x} - \boldsymbol{y}\|_2^2 + 2\left\|\boldsymbol{z} - \frac{1}{2}(\boldsymbol{x} + \boldsymbol{y})\right\|_2^2.$$

3. Let $\boldsymbol{x}, \boldsymbol{y}$ be vectors in $\mathbb{R}^n$ such that $\boldsymbol{x}^T\boldsymbol{y} = 0$. Prove the *Pythagorean theorem*
$$\|\boldsymbol{x} - \boldsymbol{y}\|_2^2 = \|\boldsymbol{x}\|_2^2 + \|\boldsymbol{y}\|_2^2.$$

4. Do Exercise 5.2.6 in the textbook.

5. Do Exercise 5.2.7 in the textbook.