

Math 532 — Homework 8 — Due: Wednesday, March 11, 2015

1. Consider a general (complex) inner product space \mathcal{V} and show that the following two properties follow from the basic definition:

$$\begin{aligned}\langle \alpha \mathbf{x}, \mathbf{y} \rangle &= \bar{\alpha} \langle \mathbf{x}, \mathbf{y} \rangle \\ \langle \mathbf{x} + \mathbf{y}, \mathbf{z} \rangle &= \langle \mathbf{x}, \mathbf{z} \rangle + \langle \mathbf{y}, \mathbf{z} \rangle.\end{aligned}$$

2. Prove that the $p = \infty$ vector norm on \mathbb{R}^n , $n \geq 2$ is not induced by an inner product.
3. Show that the reduced QR factorization — as defined in the notes — is unique.
4. Consider two Householder reflections R_1 and R_2 .
 - (a) Show that $\begin{pmatrix} R_1 & O \\ O & R_2 \end{pmatrix}$ can't be another Householder reflection.
 - (b) Show that $\begin{pmatrix} I & O \\ O & R_2 \end{pmatrix}$ is a Householder reflection.
5. Do Exercise 5.5.9 in the textbook.