Math 532 — Pre-Test — Monday, January 12, 2015

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1. Construct a 3×4 matrix A and 3×1 column vectors $\boldsymbol{b}$ and $\boldsymbol{c}$ such that the *augmented matrix* $\begin{pmatrix} \mathsf{A} & \boldsymbol{b} \end{pmatrix}$ is the augmented matrix for an inconsistent linear system $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$, but $\begin{pmatrix} \mathsf{A} & \boldsymbol{c} \end{pmatrix}$ is the augmented matrix for a consistent system.

2. Assume that $m < n$ are positive integers and explain why a homogeneous linear system $\mathsf{A}\boldsymbol{x} = \mathbf{0}$ of m equations in n unknowns must always possess infinitely many solutions.

3. Consider the linear system $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$ with

$$\mathsf{A} = \begin{pmatrix} 2 & 2 & 3 \\ 4 & 8 & 12 \\ 6 & 2 & \alpha \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} 0 \\ -4 \\ 4 \end{pmatrix}.$$

   (a) Determine all values of α for which the system is *consistent*, i.e., for which the system has at least one solution.

   (b) Determine all values of α for which there is a unique solution, and compute the solutions for these cases.

   (c) Determine all values of α for which there are infinitely many solutions, and give the general solution for these cases.

4. The *trace* of a square matrix A is given by the sum of its diagonal elements, i.e.,

$$\operatorname{trace}(\mathsf{A}) = \sum_{i=1}^{n} [\mathsf{A}]_{ii}.$$

   Show that $\operatorname{trace}(\cdot)$ is a linear function.

5. Let A be an $m \times n$ matrix. The *transpose* A^T of A is defined by $[\mathsf{A}^T]_{ij} = [\mathsf{A}]_{ji}$. Use this definition to prove that

$$(\mathsf{A} + \mathsf{B})^T = \mathsf{A}^T + \mathsf{B}^T$$

   provided both A and B are $m \times n$ matrices.

6. Prove or disprove: If A, B and C are matrices of compatible sizes, then $\mathsf{AB} = \mathsf{AC}$ implies $\mathsf{B} = \mathsf{C}$.

7. Let

$$\mathsf{E} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

   and let A be an arbitrary 3×3 matrix.

   (a) Describe the rows of EA in terms of the rows of A.

   (b) Describe the columns of AE in terms of the columns of A.

8. For all $n \times k$ matrices A and $k \times n$ matrices B, show that the block matrix

$$\mathsf{L} = \begin{pmatrix} \mathsf{I} - \mathsf{BA} & \mathsf{B} \\ 2\mathsf{A} - \mathsf{ABA} & \mathsf{AB} - \mathsf{I} \end{pmatrix}$$

   satisfies $\mathsf{L}^2 = \mathsf{I}$, and therefore is a so-called *involutory* matrix.