MATH 100 TOPIC
THE EXPONENTIAL FUNCTION IN FINANCE
(CHAPTER 8 OF *EXPERIMENTS WITH MATLAB*)

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The differential equation $y' = ry$, $y(0) = 1000$, describes how your initial \$1000 invested with a constant return grows with time. Think of r as the return on investment, say, the percentage rate on a certificate of deposit or the rate of interest on a savings account.

1. If $r = 2\%$, how much have you earned in 10 years?

Now suppose that every quarter (three months), interest rates are revised by the bank and they increase by 0.25% with a probability of $p \leq 0.5$, decrease by 0.25% with a probability of p, or remain unchanged with a probability of $1 - 2p$.

2. What happens to your investment now over a time horizon of 10 years?

Construct a model for $y(t)$ as a function of $r(t)$, where $r(t)$ varies randomly as described above. You will need to consider random numbers and write a MATLAB program that generates many possible scenarios for how r, and thus y, change with time. Based on a taking the average of a good number of scenarios on estimate the average that you can expect to earn, and how it depends on p.

If you wish, consider an investment in a stock that may go up or down in value due to market forces. How do you modify the initial differential equation to describe this case, and how do you solve it with the aid of MATLAB?