MATH 100 – Introduction to the Profession

Applied Mathematicians – Who are we, and what do we do?

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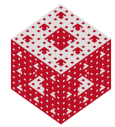

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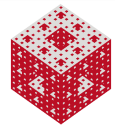
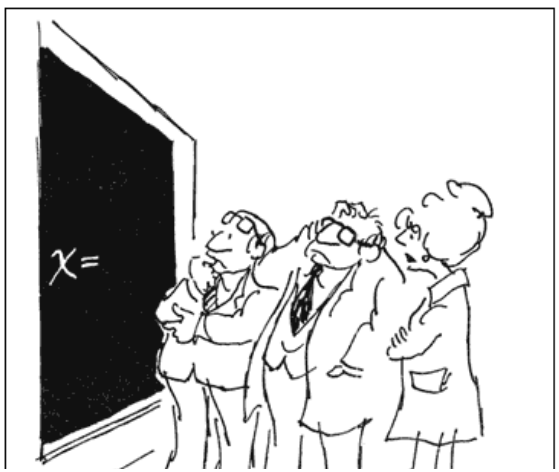


What is Mathematics About?

$\mu\acute{\alpha}\theta\eta\mu\alpha$ (máthēma): knowledge, study, learning

It's not just

- about numbers
- about solving for x
- the "science of quantity and space" (arithmetic and geometry)



It is about

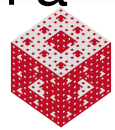
- patterns
- structure
- abstraction
- models
- rigor
- many aspects of science and phil

Mathematicians

According to Alfréd Rényi:

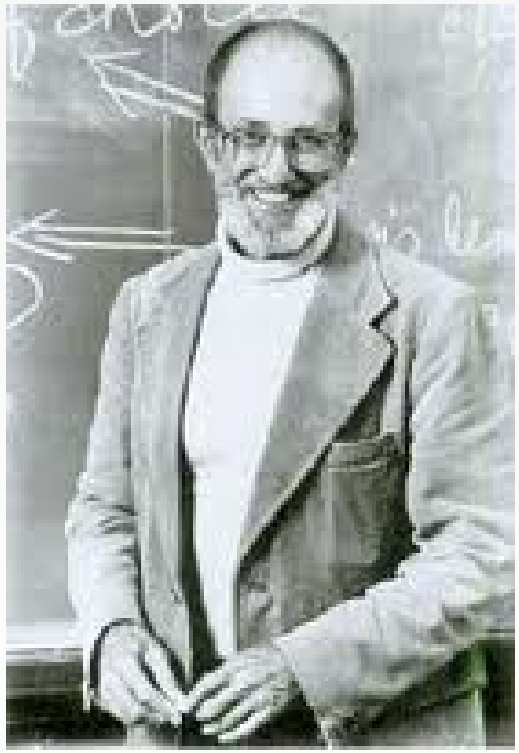
A mathematician is a machine that turns coffee into theorems.

And – speaking of coffee – there's:
A topologist is a mathematician who can't tell the difference between a doughnut and a coffee mug.



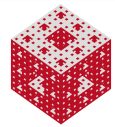
"What does mathematics *really* consist of?

- Axioms (such as the parallel postulate)?
- Theorems (such as the fundamental theorem of algebra)?
- Proofs (such as Gödel's proof of undecidability)?
- Definitions (such as the Menger definition of dimension)?
- Theories (such as category theory)?
- Formulas (such as Cauchy's integral formula)?
- Methods (such as the method of successive approximations)?



Mathematics could surely not exist without these ingredients; they are all essential. It is nevertheless a tenable point of view that none of them is at the heart of the subject, that the mathematician's main reason for existence is to solve problems, and that, therefore, what mathematics *really* consists of is problems and solutions."

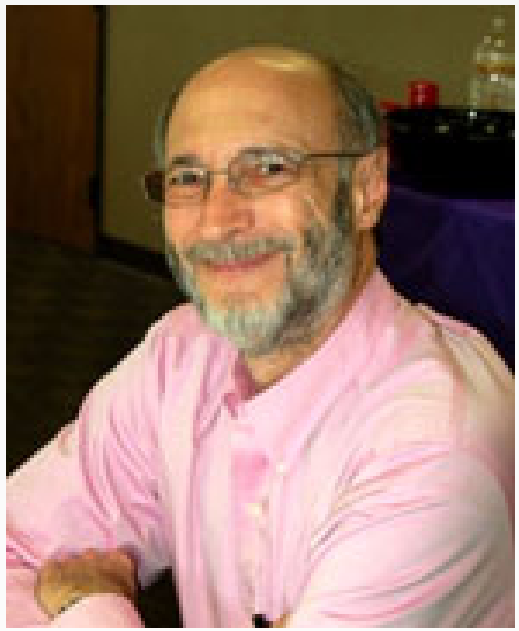
Paul Halmos [The heart of mathematics]

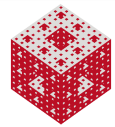


"Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns – systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically ('pure mathematics') or models of systems abstracted from real world objects ('applied mathematics').

The tools of mathematics are abstraction, symbolic representation, and symbolic manipulation."

Alan Schoenfeld [Learning to think mathematically]





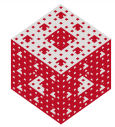
Mathematicians

According to Steven Krantz [Krantz4] (who gives credit to Keith Devlin), a mathematician is someone who:

- Observes and interprets phenomena.
- Analyzes scientific events and information.
- Formulates concepts.
- Generalizes concepts.
- Performs inductive reasoning.
- Performs analogical reasoning.
- Engages in trial and error (and evaluation).
- Models ideas and phenomena.
- Formulates problems.
- Abstracts from problems.
- Solves problems.
- Uses computation to draw analytical conclusions.
- Makes deductions.
- Makes guesses.
- Proves theorems.

A mathematician is a master of critical thinking, of analysis, and of deductive reasoning.

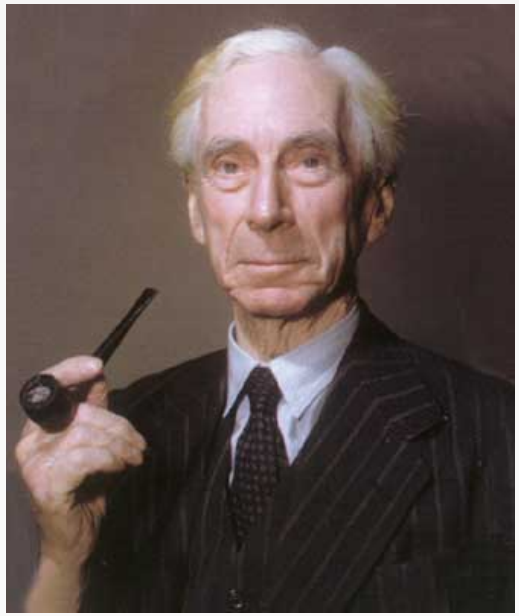
Famous mathematicians: (e.g., `http://en.wikipedia.org/wiki/Mathematician`)



Beautiful Mathematics

"Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."

(Bertrand Russell, The Study of Mathematics)



Example

Euler's formula:

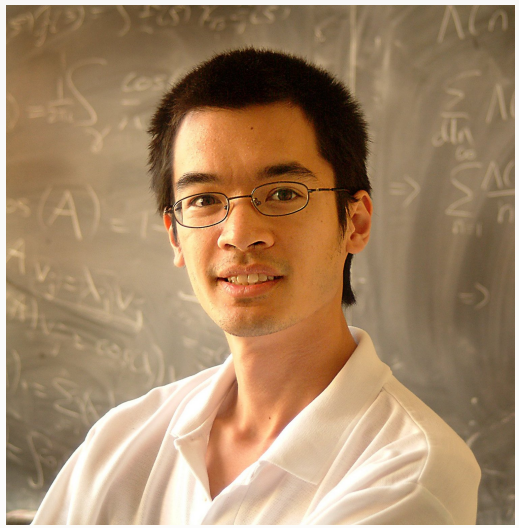
$$e^{i\pi} + 1 = 0$$

considered by many "the most beautiful theorem in mathematics".

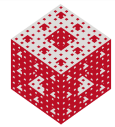
Good Mathematics

We all agree that mathematicians should strive to produce good mathematics. But how does one define "good mathematics", and should one even dare to try at all? Let us first consider the former question. Almost immediately one realises that there are many different types of mathematics which could be designated "good". For instance, "good mathematics" could refer (in no particular order) to

(Terence Tao, [What Is Good Mathematics?])



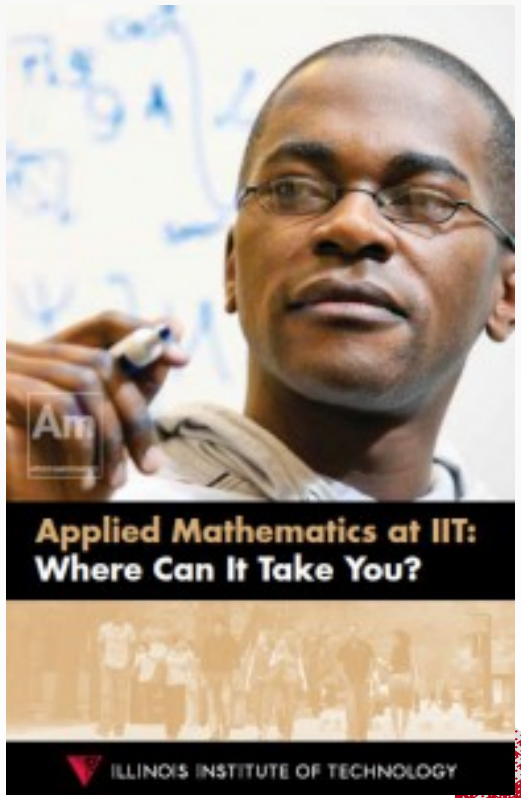
see Terry Tao's list here.



Pure Math vs. Applied Math – A never-ending story

From Applied Mathematics at IIT: `http://blogs.iit.edu/choose_your_major/files/2011/07/AM_Bro_WEB-2.pdf`

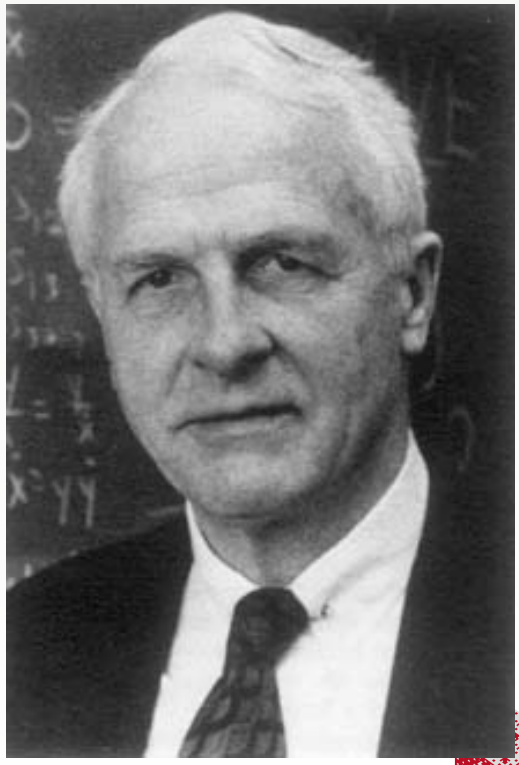
Mathematics is an elegant, precise, and rigorous way of solving problems involving things that can be measured. Applied mathematicians develop new mathematical tools and utilize them to address problems arising in all aspects of society. For example, our faculty members take part in efforts to detect national security threats in communication networks, to predict the weather more reliably, to manage financial risk, to understand cell and tumor growth, and to design safer nuclear reactors.



More Pure vs. Applied

A comment by Garrett Birkhoff (son of George David Birkhoff, IIT-Lewis College, 1896-1902) in [Applied Mathematics and Its Future (1977)]:

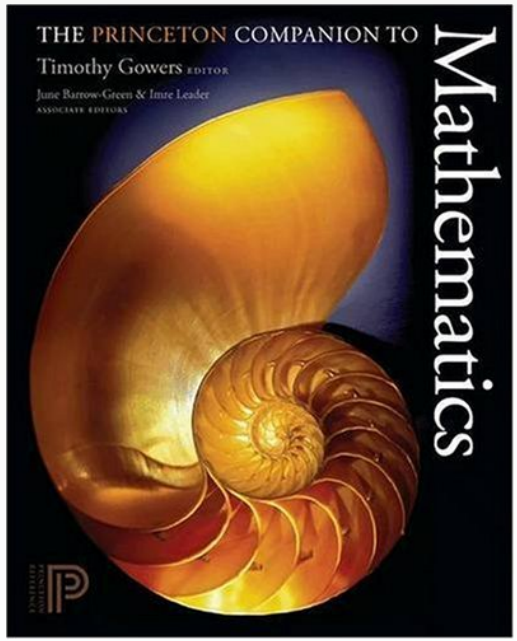
Essentially, mathematics becomes “applied” when it is used to solve real-world problems “neither seeking nor avoiding mathematical difficulties” (Rayleigh). Thus applied mathematics encompasses the *interdisciplinary aspects of mathematics*.

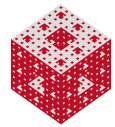


More Pure vs. Applied

From [The Princeton Companion to Mathematics]:

The word “pure” is more troublesome. As many have commented, there is no clear dividing line between pure and applied mathematics, and, just as a proper appreciation of modern mathematics requires some knowledge of its history, so a proper appreciation of pure mathematics requires some knowledge of applied mathematics and theoretical physics.

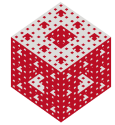




More Pure vs. Applied

Comments by Reuben Hersh in [Definition of Mathematics]:

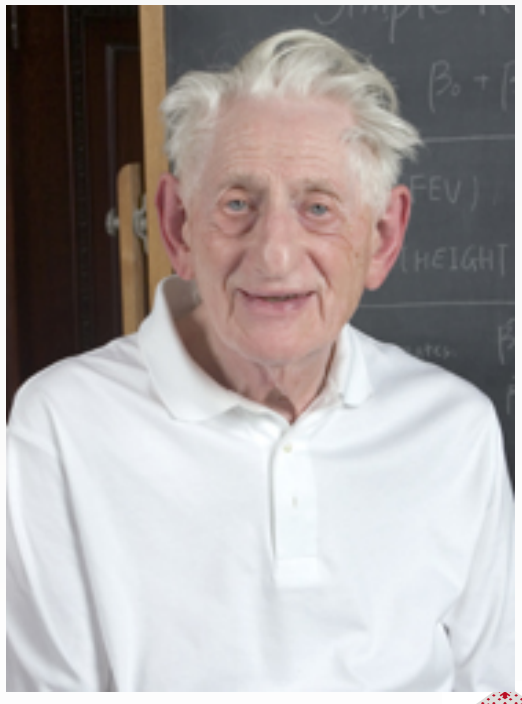
Applied mathematics uses whatever arguments and methods it can – analogy, special examples, numerical approximations, physical models – to learn about hurricanes, say, or epidemics. It is mathematical activity, to the extent that it makes use of mathematical concepts and results, which are, by definition, concepts and results capable of strict mathematical reasoning – rigorous proof.



More Pure vs. Applied

. . . and from [The Mathematical Experience] by Philip Davis and Reuben Hersh:

In the last few years, there has been a noticeable shift in the attitudes predominant among American mathematicians. Applied mathematics is becoming stylish. . . . Of the jobs one sees advertised, many call for competence in statistics, in computing, in numerical analysis or in applied mathematics.



More Pure vs. Applied

Now, some excerpts from one of the most famous works on this topic:

The 'real' mathematics of the 'real' mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Riemann, is almost wholly 'useless' (and this is as true of 'applied' as of 'pure' mathematics). It is not possible to justify the life of any genuine professional mathematician on the ground of the 'utility' of his work. . . . I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world.

(G. H. Hardy, A Mathematician's Apology)

Irony: much of Hardy's work is now being 'applied'.

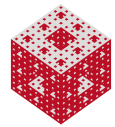
More Pure vs. Applied

Finally, comments by Nick Trefethen (president of SIAM) written April 6, 2011: `http://www.siam.org/news/news.php?id=1875`

Pure mathematicians are brilliant but clueless. Applied mathematicians don't care if something is true so long as it works. . . . Much of what applied mathematicians produce is forgotten ten years later. Most of the really top people go into pure mathematics. . . . Most pure mathematicians know little science and even less engineering. They don't care about phenomena, just technique. . . . maybe pure and applied mathematics are like classical music and rock and roll. Elton John and Maurizio Pollini are probably not regular drinking buddies, but when you come down to it, they're both colleagues in the piano business.

Professional Organizations

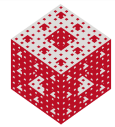
- http://www.siam.org/ (Society for Industrial and Applied Mathematics)
- http://www.ams.org/ (American Mathematical Society)
- http://www.maa.org/ (Mathematical Association of America)
- http://www.awm-math.org/ (Association for Women in Mathematics)
- http://www.amstat.org/ (American Statistical Association)
- http://www.acm.org/ (Association for Computing Machinery)
- http://www.soa.org/ (Society of Actuaries)
- http://www.informs.org/ (Institute for Operations Research and the Management Sciences)
- http://www.ieee.org/ (Institute of Electrical and Electronics Engineers)
- http://www.nctm.org/ (National Council of Teachers of Mathematics)



Often these organizations have free student membership.

They provide lots of benefits such as reduced conference fees, cheaper books, journals, etc.

And they are a great source for career information (see the info compiled at `http://math.iit.edu/~fass/100.html`)

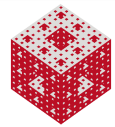


Problem Solving

The crucial four phases according to George Polya (see `http://www.math.utah.edu/~pa/math/polya.html`):

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

Alternate resource: Steven Krantz' [Techniques of Problem Solving].
We will discuss mathematical models later (Chapter 1 of Gowers)

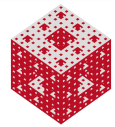


More Problem Solving

How to tackle “new” and “challenging”, previously unexplored, problems:

- Read the problem carefully. Restate the problem to possibly uncover more information.
- Think about a few similar, but simpler, problems and investigate them first. Try some simple cases (with numbers). Try to identify a pattern.
- Use what you learned from your initial investigations to come up with a strategy to tackle the more complex problem.
- Break your overall strategy down into smaller steps. “If I know how to get this far, then . . .” (think backwards).
- Consider which mathematical tools you need for each of the steps.
- Be creative.

Look at the example discussed in class.



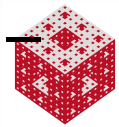
Communication: Mathematical Writing

- Use a similar approach to communicating your work as you did to solve your problem.
- Consider the "big picture" and then talk about crucial steps.

"Good writing . . . is clear thinking made visible".
(Ambrose Bierce, Write it Right)

Some quotes from [Higham]:

- Writing helps you to learn:
  - Writing brings out gaps in your understanding, by forcing you to focus on steps in your thinking that you might otherwise skip.
- Good writing reflects clear thinking.
- Writing is difficult!
- Keep your prose simple and direct.
- One way to improve your writing is to read as much as you can – especially by "good" writers ([Higham] lists many examples).



Writing Resources

- Some guides to mathematical writing are [Higham, Knuth, Krantz2]
- "How to Read Mathematics" [How to Read]
- "How to Write a Solution" [How to Write] (many other resources there as well)
- Most mathematicians use LaTeX (see the info compiled at `http://math.iit.edu/~fass/100.html`)

Other Survival Skills

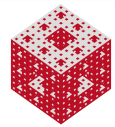
Common problems today:

- Students feel “overwhelmed”
- They have trouble coping emotionally
- Their life is not “falling into place” quick enough

$\longrightarrow$ “Generation WTF”

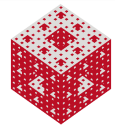
Suggestions for improvement from
[Helping First-Year Students Help Themselves]:

- Have clear goals early, and follow the steps to get there (see the tools, quizzes and other resources at `http://generationwtf.com/`)
- Work on your communications skills
- Develop the ability to make decisions



Other Good Ideas

- Make friends with other students – also older ones (e.g., join the Math Club)
- Organize/join a study group
- Ask for help: other students, TAs, professors, ARC
- Talk to your professors (also about non-academic things)
- Get involved on campus, e.g., join a student organization or club



References I

In addition to our official textbooks and the links given earlier, you may want to look at some of the following resources.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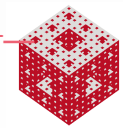

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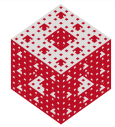

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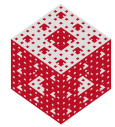

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