MATH 100 – Introduction to the Profession

Matrices and Linear Transformations in MATLAB

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Basic Definitions of Vectors and Matrices

We go over some basic matrix and vector stuff in MATLAB:

- `matrix_vector.m` (definition of matrices and vectors)
- `arithmetic.m` (simple arithmetic with matrices and vectors)
- `lin_sys.m` (solving linear systems)
- `submatrices.m` (definition of submatrices)

Vector Equation of a Line

An arbitrary point $\boldsymbol{r}$ on a line through the point $\boldsymbol{r}_0$ with direction vector $\boldsymbol{v}$ is given by

$$\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v},$$

where the parameter t tells us how much of, and which direction, the vector $\boldsymbol{v}$ is added to $\boldsymbol{r}_0$.

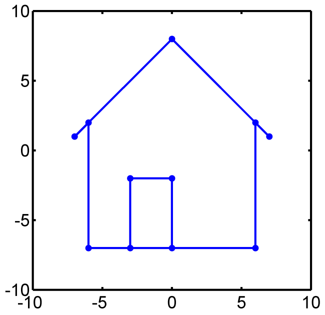
Look at the Mathematica demo `EquationOfALineInVectorForm2D.cdf`.

More details on vectors and equations of lines in 2D and 3D are given in [Stewart Calculus, Sections 12.2 and 12.5].

Matrices as Linear Transformations

We illustrate properties of linear transformations (matrix multiplication by A) with the following “data”:

```
X = house
dot2dot(X)
```

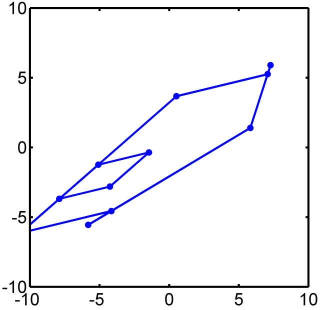


Straight lines are always mapped to straight lines.

```
A = rand(2,2)
dot2dot(A*X)
```

Sample matrix

$$A = \begin{bmatrix} 0.9357 & 0.7283 \\ 0.8187 & 0.1758 \end{bmatrix}$$

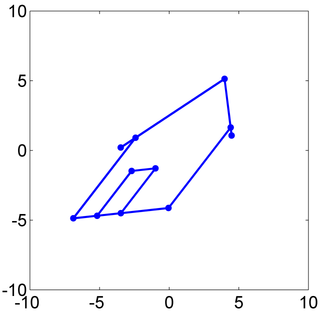


The transformation is orientation-preserving[1] if $\det A > 0$.

```
A = rand(2,2)
det(A)
dot2dot(A*X)
```

Sample matrix

$$A = \begin{bmatrix} 0.5694 & 0.4963 \\ 0.0614 & 0.6423 \end{bmatrix}$$



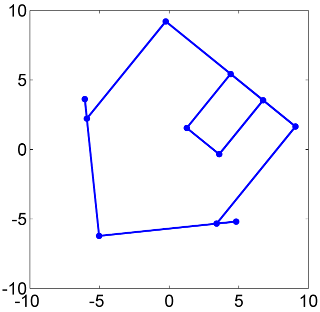
[1] The door always stays on the left.

The angles between straight lines are preserved if the matrix is orthogonal[2].

```
A = orth(rand(2,2));   % creates orthogonal matrix
A = A(:,randperm(2))   % randomly permute columns of A
det(A)
dot2dot(A*X)
```

Sample matrix

$$A = \begin{bmatrix} -0.7767 & -0.6299 \\ 0.6299 & -0.7767 \end{bmatrix}$$



[2] An orthogonal matrix A has $\det A \pm 1$ and represents either a rotation or a reflection.

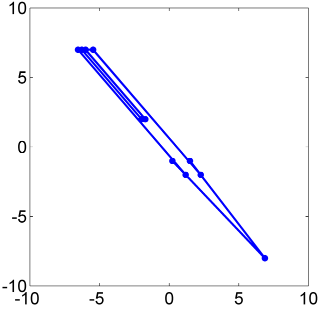
A linear transformation is invertible[3] only if $\det A \neq 0$.

```
a22 = randi(3,1,1)-2  % creates random {-1,0,1}
A = triu(rand(2,2)); A(2,2) = a22
det(A)
dot2dot(A*X)
```

Sample matrix

$$A = \begin{bmatrix} 0.0903 & 0.8586 \\ 0 & -1.0000 \end{bmatrix}$$

$$\det A = -0.0903$$



[3] If the transformation is not invertible, then the 2D image collapses to a line or even a point.

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```
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A = triu(rand(2,2)); A(2,2) = a22
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```
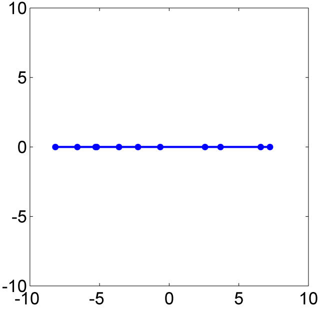

Sample matrix

$$A = \begin{bmatrix} 0.9884 & 0.3209 \\ 0 & 0 \end{bmatrix}$$

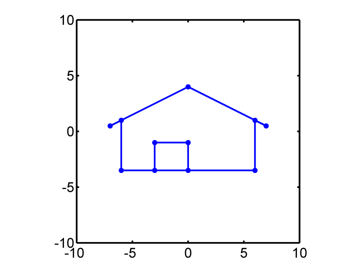
$$\det A = 0$$



[3]If the transformation is not invertible, then the 2D image collapses to a line or even a point.

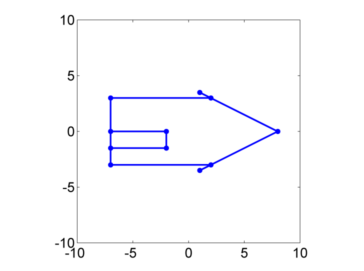
A diagonal matrix stretches the image or reverses its orientation.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \quad \det A = \frac{1}{2}$$



A anti-diagonal matrix in addition interchanges coordinates.

$$A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & 0 \end{bmatrix}, \quad \det A = -\frac{1}{2}$$



The action of a diagonal matrix provides an interpretation of the effect of eigenvalues. Note that these matrices have orthogonal columns, but their determinant is not ± 1, so they are **not** orthogonal matrices. These matrices preserve right angles only.

Any rotation matrix can be expressed in terms of trigonometric functions:
The matrix

$$G(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

represents a counter-clockwise rotation by the angle θ (measured in radians).

Look at `wiggle.m`.

Look through `matrices_recap.m`.

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